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Democratic-type neutrino mass matrix ^{*}

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Abstract

We consider the democratic-type neutrino mass matrix and show that this matrix predicts the atmospheric neutrino mixing to be almost maximal, $\sin^2 2\theta_{atm} > 0.999$ as well as the large CP violation (the CP violation phase in the standard form is maximal $\delta = \pi/2$). We construct the Z_3 symmetric dimension five effective Lagrangian with two up-type Higgs doublets and show that this Lagrangian leads to the democratic neutrino mass matrix. Furthermore, we consider the restricted model with one up-type Higgs doublet and obtain the prediction, $0.87 < \sin^2 2\theta_{sol} < 8/9$.

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1 Introduction

The observation of the atmospheric neutrino by SuperKamiokande[1] has shown the existence of the neutrino masses and the neutrino mixings. In particular, the data show that the mixing between ν_μ and ν_τ is favored and $\sin^2 2\theta_{atm} \simeq 1$ and $\Delta_{atm}^2 \sim 3.5 \times 10^{-3} \text{eV}^2$. The solar neutrino problem is now considered to be due to the ν_e and ν_μ oscillation, but the information on masses and mixing angles are ambiguous. Now four solutions are given[2]. The another crucial information is given by CHOOZ group[3] that $|V_{13}| < 0.16$

We interpret these in the neutrino mixing matrix in the standard form[4]

$$V_{SF} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (1)$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$.

The large atmospheric neutrino mixing requires $|s_{23}| \simeq |c_{23}| \simeq 1/\sqrt{2}$ and the CHOOZ data gives the bound $|s_{13}| < 0.16$.

Now we face the following questions: (1) Why $|s_{23}| \sim |c_{23}| \sim 1/\sqrt{2}$? (2) What is the size of s_{12} ? (3) Why s_{13} is so small? (4) What is the size of the CP violation phase δ ?

It is quite hard to construct the neutrino mass matrix which answers all these questions. In this note, we focus on the questions, (1), (2) and (4), by using the democratic-type mass matrix for the neutrino[5],[6]. Throughout of this note, we consider the neutrino mass matrix in the basis where charged leptons mass matrix is diagonal.

2 The model construction

We are interested in the CP violation phase. A famous model that predicts the CP violation phase is the Tri-maximal mixing scheme[7]

$$V_T = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix}, \quad (2)$$

where ω is the element of Z_3 and is given by $\omega = e^{i2\pi/3}$, i.e., $\omega^3 = 1$. This matrix predicts $|V_{13}| = 1/\sqrt{3}$ in conflict with the CHOOZ data[3]. However, this scheme has an quite interesting predictions; (i) The maximal CP phase, $\delta = \pi/2$ and (ii) the maximal CP violation, $|J_{CP}|_{Tri} = 1/6\sqrt{3}$.

(a) Deformation from the Tri-maximal mixing

We considered the deformation of the Tri-maximal mixing matrix by the orthogonal matrix O , $V = V_T O$ and found quite significant predictions[5],

$$|s_{23}| = |c_{23}| = \frac{1}{\sqrt{2}}, \quad \delta = \frac{\pi}{2}. \quad (3)$$

(b) The possible origin of the mixing matrix in the form of $V = V_T O$

We define the new basis (ψ basis) which is related to the flavor eigenstate basis by $(\psi_1, \psi_2, \psi_3)^T = V_T^\dagger (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^T$. Now, the matrix to realize the mixing matrix $V = V_T O$ is some real matrix in the ψ basis,

$$V_T^T m_\nu V_T = \begin{pmatrix} m_1^0 + \tilde{m}_1 & \tilde{m}_3 & \tilde{m}_2 \\ \tilde{m}_3 & m_2^0 + \tilde{m}_2 & \tilde{m}_1 \\ \tilde{m}_2 & \tilde{m}_1 & m_3^0 + \tilde{m}_3 \end{pmatrix}, \quad (4)$$

where m_i^0 and \tilde{m}_i are real parameters. By inverting, we obtain m_ν in the flavor eigenstate basis as

$$\begin{aligned} m_\nu = & \frac{m_1^0}{3} \begin{pmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{pmatrix} + \frac{m_2^0}{3} \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + \frac{m_3^0}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \\ & + \tilde{m}_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} + \tilde{m}_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} + \tilde{m}_3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \end{aligned} \quad (5)$$

which we called the democratic-type mass matrix[5].

(c) The dimension five effective Lagrangian with the Z_3 symmetry

The democratic-type mass matrix is derived from the Lagrangian by imposing the Z_3 symmetry. We define the irreducible representations of Z_3 as $\Psi_1 = \frac{1}{\sqrt{3}}(\ell_e + \omega^2 \ell_\mu + \omega \ell_\tau)$, $\Psi_2 = \frac{1}{\sqrt{3}}(\ell_e + \omega \ell_\mu + \omega^2 \ell_\tau)$ and $\Psi_3 = \frac{1}{\sqrt{3}}(\ell_e + \ell_\mu + \ell_\tau)$, where ℓ_i is the left-handed lepton

doublet defined by, say, $\ell_e^T = (\nu_{eL}, e_L)$. The fields Ψ_i transform under the permutation ℓ_e, ℓ_μ and ℓ_τ , $\Psi_1 \rightarrow \omega \Psi_1$, $\Psi_2 \rightarrow \omega^2 \Psi_2$ and $\Psi_3 \rightarrow \Psi_3$. With the definition, $\Psi_i = (\psi_i, e_i)$, the above relation is viewed as the transformation from the flavor eigenstate basis to the ψ basis.

If we introduce two up-type Higgs doublets that behave as $H_{u1} \rightarrow \omega^2 H_{u1}$ and $H_{u2} \rightarrow \omega H_{u2}$, then we can construct the Z_3 invariant dimension five effective Lagrangian as

$$\begin{aligned} \mathcal{L}_y = & - \left((m_1^0 + \tilde{m}_1) \overline{(\Psi_1)^C} \Psi_1 \frac{H_{u1} H_{u1}}{u_1^2} + (m_2^0 + \tilde{m}_2) \overline{(\Psi_2)^C} \Psi_2 \frac{H_{u2} H_{u2}}{u_2^2} \right. \\ & \left. + (m_3^0 + \tilde{m}_3) \overline{(\Psi_3)^C} \Psi_3 \frac{H_{u1} H_{u2}}{u_1 u_2} \right) \\ & - 2 \left(\tilde{m}_1 \overline{(\Psi_2)^C} \Psi_3 \frac{H_{u1} H_{u1}}{u_1^2} + \tilde{m}_2 \overline{(\Psi_1)^C} \Psi_3 \frac{H_{u2} H_{u2}}{u_2^2} + \tilde{m}_3 \overline{(\Psi_1)^C} \Psi_2 \frac{H_{u1} H_{u2}}{u_1 u_2} \right), \quad (6) \end{aligned}$$

where u_i is the vacuum expectation value of the neutral component of H_{ui} .

After the Higgs fields acquire the vacuum expectation values, the neutrino mass matrix in the ψ -basis defined by Eq.(4), and thus the democratic-type neutrino mass matrix in Eq.(5) is obtained.

3 A restricted model -one Higgs case-

Here we consider the case with only one up-type Higgs by keeping H_{u1} . Then, from Eq.(6), we only have the \tilde{m}_1 and m_1^0 terms. As we can see from Eq.(4) with $\tilde{m}_2 = \tilde{m}_3 = 0$ and $m_2^0 = m_3^0 = 0$, we observe that the matrix is diagonalized by

$$V_1 = V_T \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & i \end{pmatrix}.$$

This mixing matrix is a new type and predicts $\sin^2 2\theta_{sol} = \frac{8}{9}$ and $\sin^2 2\theta_{atm} = 1$ in contrast to the Bi-maximal mixing[8] which predicts $\sin^2 2\theta_{sol} = 1$ and $\sin^2 2\theta_{atm} = 1$, and the democratic mixing[9] does $\sin^2 2\theta_{sol} = 1$ and $\sin^2 2\theta_{atm} = 8/9$.

Unfortunately, this model predicts the degenerate masses $m_2 = -m_3 (= \tilde{m}_1)$. In order to remedy this deficit, we introduce the symmetry breaking terms which preserve the

Z_2 symmetry $\Psi_1 \rightarrow -\Psi_1$. Among the interaction terms in Eq.(14), the Z_2 symmetry excludes \tilde{m}_2 and \tilde{m}_3 terms and thus we obtain the neutrino mass matrix including four parameters \tilde{m}_1 , m_1^0 , m_2^0 and m_3^0 . This matrix is diagonalized by[6]

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}}c' & i\sqrt{\frac{2}{3}}s' \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}}(c' + i\sqrt{3}s') & -\frac{1}{\sqrt{6}}(\sqrt{3}c' + is') \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}}(c' - i\sqrt{3}s') & -\frac{1}{\sqrt{6}}(\sqrt{3}c' - is') \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & i \end{pmatrix}, \quad (7)$$

where $s' = \sin \theta'$, $c' = \cos \theta'$ and $\tan \theta' = \Delta_- / (\tilde{m}_1 + \sqrt{\tilde{m}_1^2 + \Delta_-^2})$. The neutrino masses are given by $m_1 = m_1^0 + \tilde{m}_1$, $m_2 = m_2^0 + \Delta_- + \sqrt{\tilde{m}_1^2 + \Delta_-^2}$, $m_3 = m_2^0 + \Delta_- - \sqrt{\tilde{m}_1^2 + \Delta_-^2}$.

From the mixing matrix in Eq.(7), we find $\delta = \frac{\pi}{2}$ and $\tan \theta_{12} = \sqrt{2 - 3 \sin^2 \theta_{13}}$, while s_{13} is left a free parameter. Now we have $\sin^2 2\theta_{sol} = \frac{8}{9}c'^2$ and $\sin^2 2\theta_{atm} = 1 - \frac{4}{9}s'^4$. If we impose the CHOOZ bound, $|\sqrt{2/3}s'| < 0.16$, we have $0.87 < \sin^2 2\theta_{sol} < 8/9$ and $\sin^2 2\theta_{atm} > 0.999$. If the future precision experiments on the neutrino mixing show that the atmospheric neutrino mixing is very close to the maximal value and the solar neutrino mixing is large but not maximal, our model would be a very good candidate.

Here, the phase matrix $\text{diag}(1, \omega, \omega^2)$ is absorbed into charged lepton phases and the phase matrix $\text{diag}(1, -1, i)$ represents the CP violating Majorana phase matrix[10],[11], which is relevant to the purely lepton number violating processes such as the neutrinoless double beta decay[12].

As for the CP violation, the Jarlskog parameter is

$$\frac{|J_{CP}|_{our\ model}}{|J_{CP}|_{max}} = \sqrt{6} \left| s_{13} c_{13}^2 \sqrt{1 - \frac{3}{2}s_{13}^2} \right| < 0.37, \quad (8)$$

where $|s_{13}| = \sqrt{2/3}s'$. The reduction rate from the maximum CP violation is solely dependent on the mixing angle s_{13} . If we use the CHOOZ data $|s_{13}| < 0.16$, we obtain the ratio is smaller than 0.37, which is good enough to be observed in the near future laboratory experiments on the neutrino oscillations.

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